Saguaro East Best Fit Solutions

Phillip McFarland
for George H. Davis

Method:

Three sets of GPS data were analyzed. The data sets were given in UTM UPS, with corresponding elevations given in meters above sea level. The goal of the analysis was to assign a plane of best fit to each of the three data sets. All of the data points were contained within the UTM quadrant designated 12S. Therefore, for convenience of converting the UTM/elevation data to cartesian coordinates, the southwestern most point of 12S and 0 meters above sea level was chosen as the origin for the cartesian reference frame. For example, the point given by $(526836 \text{ mE}, 3561480 \text{ mN}, 933$ meters above sea level) would correspond to the cartesian triple $(526836, 3561480, 933)$. Using these cartesian triples, a least squares approach was used to find a plane of best fit.

The equation for a plane in cartesian coordinates is given as:

$$Ax + By + Cz + D = 0 \tag{1}$$

Rearranging this gives:

$$z = -\frac{A}{C}x - \frac{B}{C}y - \frac{D}{C} \tag{2}$$

Letting $\alpha = -\frac{A}{C}$, $\beta = -\frac{B}{C}$ and $\eta = -\frac{D}{C}$ equation 2 becomes:

$$z = \alpha x + \beta y + \eta \tag{3}$$

Plugging the cartesian triples into equation 3 for x, y and z gives:

$$\begin{aligned} z_1 &= \alpha x_1 + \beta y_1 + \eta \\ z_2 &= \alpha x_2 + \beta y_2 + \eta \\ \vdots & \vdots \vdots \\ z_n &= \alpha x_n + \beta y_n + \eta \end{aligned}$$

Rewriting this system of equations in matrix form gives:

$$\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots \\ x_n & y_n & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \eta \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix} \tag{4}$$

In matrix notation equation 4 is rewritten as:

$$A\vec{e} = \vec{z} \tag{5}$$

Equation 4/5 is overdetermined. In order to find a least squares solution the usual method is used:

$$\hat{e} = \left(A^T A\right)^{-1} A^T \vec{z} \tag{6}$$

Where, $\hat{e}$ in equation 7 is an approximation of $\vec{e}$. MatLab was used to compute A, A^T, $\left(A^T A\right)^{-1}$ and $\hat{e}$.

Using the data set for the ultramylonite a least squares estimate ($\hat{e_u}$) was calculated to be:

$$\hat{e_u} = \begin{bmatrix} 0.1098 \\ -0.0651 \\ 1.7485 \times 10^5 \end{bmatrix} \tag{7}$$

So, the equation of the best fit plane for the ultramylonite data is written as:

$$z_u = 0.1098x_u + -0.0651y_u + 1.7485 \times 10^5 \tag{8}$$

To communicate the meaning of this to a structural geologist it is useful to convert the orientation of the plane from cartesian coordinates to "strike" and "dip". The strike of the plane is given by the compass orientation of any line along which the value of z (elevation) does not change. In order to compute the orientation of this line one must choose some constant value of z. The easiest value to work with is $z = 0$. Plugging $z = 0$ into equation 8 gives:

$$y_u = 1.686635945x_u + 2.685867896 \times 10^6 \tag{9}$$

Equation 9 is in slope-intercept form. So, the angle away from the $y_u - axis$ (θ_u) will be found by computing:

$$\theta_u = 90^\circ - \tan^{-1}(1.686635945) \tag{10}$$

$$\theta_u = 30.66^\circ \tag{11}$$

But, θ_u could be the orientation of the strike line or it might be $180^\circ + \theta_u$. In order to determine which it is, the dip-direction must be calculated. Taking the gradient of the equation for the plane of best fit will generate a vector in the direction of greatest elevation change along the plane. Taking the negative gradient will generate a vector in the direction of greatest negative elevation change along the plane. This direction is the dip direction. The negative gradient ($-\nabla z_u$) is calculated to be:

$$-\nabla z_u = \langle -0.1098, 0.0651 \rangle \tag{12}$$

This means that the dip-direction is to the northwest.

In order to calculate the dip-angle two vectors were defined:

$$\vec{u_u} = \langle -0.1098, 0.0651, 0 \rangle \tag{13}$$

$$\vec{v_u} = \langle -0.1098, 0.0651, -0.01629405 \rangle \tag{14}$$

$\vec{u_u}$ is the vector in the direction of greatest negative elevation change along the plane of best fit. $\vec{v_u}$ is the vector in the same direction as $\vec{u_u}$ but with the third component as the negative elevation change in that direction. Defining $\vec{u_u}$ and $\vec{v_v}$ in this way means that the dip-angle (ϕ_u) is given by:

$$\phi_u = \cos^{-1}\left\{\frac{\vec{u_u} \cdot \vec{v_u}}{|\vec{u_u}|\,|\vec{v_u}|}\right\} \tag{15}$$

The dip-angle was calcualted to be:

$$\phi_u = 7.27^\circ \tag{16}$$

Results:

Using this method all three data sets were analyzed and the following results were determined.

Ultramylonite:

$$\begin{aligned} \text{strike} &: \text{S}30.66^\circ\text{W} \\ \text{dip} &: 7.27^\circ \text{ to the NW} \end{aligned}$$

Catalina Fault:

$$\begin{aligned} \text{strike} &: \text{S}22.89^\circ W \\ \text{dip} &: 7.32^\circ \text{ to the NW} \end{aligned}$$

Javalina Fault:

$$\begin{aligned} \text{strike} &: \text{S}26.14^\circ\text{W} \\ \text{dip} &: 5.52^\circ \text{ to the NW} \end{aligned}$$